

Vacuum Stability in Split Susy and Little Higgs Models

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Abstract

We study the stability of the effective Higgs potential in the split supersymmetry and Little Higgs models. In particular, we study the effects of higher dimensional operators in the effective potential on the Higgs mass predictions. We find that the size and sign of the higher dimensional operators can significantly change the Higgs mass required to maintain vacuum stability in Split Susy models. In the Little Higgs models the effects of higher dimensional operators can be large because of a relatively lower cut-off scale. Working with a specific model we find that a contribution from the higher dimensional operator with coefficient of $O(1)$ can destabilize the vacuum.

1 Introduction

In the standard model (SM), the electroweak symmetry breaking is achieved through a complex fundamental Higgs scalar. The discovery of the Higgs is one of the chief goals of the upcoming LHC experiment. Arguments of "triviality" [1] and "naturalness" [2], suggest that the simple spontaneous symmetry breaking mechanism in the SM may not be the whole story. In the SM, the Higgs mass receives quadratically divergent quantum corrections which has to be canceled by some new physics(NP) to obtain a sensible Higgs mass. The Higgs sector of the standard model is therefore an effective theory valid below some cut-off scale Λ . For the cancellation of the mass divergence to be not finely tuned one would require the NP scale Λ to be $\sim \text{TeV}$. One of the popular models of NP is supersymmetry(SUSY) and the phenomenology of low energy SUSY—that is SUSY broken at a scale $\sim \text{TeV}$, has been an active area of research for a long time now. However, there is no evidence of SUSY yet. As various experiments increasingly constrain the parameter space of the minimal models of low energy SUSY such as MSSM it is possible that SUSY, if present in nature, may manifest itself in experiments in a manner very different from what has been expected so far. An interesting model of SUSY with very different phenomenology than the usually studied SUSY models was recently proposed in Ref. [3, 4], which was coined the name split supersymmetry in Ref[5]. In this scenario, the Higgs mass is finely tuned to be at the weak scale while all the other scalars are much heavier than the electroweak scale. The fermions in the SUSY spectrum are allowed to be light-around a TeV or less due to chiral symmetries. This scenario can provide a dark matter candidate and the possibility of observing SUSY particles at LHC remains. By making the scalars heavy FCNC effects at low energies are suppressed. However, there are some hints of new FCNC effects in B-decays[6, 7, 8, 9, 10] and if they are confirmed then Split Susy models will have to be modified to accommodate them. Nonetheless, Split Susy models have some novel features and interesting phenomenology that have been already explored in the literature[11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29].

There are also non supersymmetric NP that can cure the Higgs mass problem. In the Little Higgs models [30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42] the Higgs mass divergence is canceled by contributions from additional particles with masses around a TeV. In this scenario, the Higgs is a pseudo Nambu-Goldstone boson whose mass is protected by some global symmetry which is spontaneously broken. The Higgs acquires a mass from explicit breaking of the global symmetry.

Whatever model one considers as a candidate for beyond the SM physics, the predictions of the Higgs mass is a key prediction of that candidate model. Direct searches at LEP has put a lower bound on the Higgs mass $m_h \sim 115$ GeV [43, 44] and from a global fit to electroweak precision observables one can put an upper bound on the Higgs mass, $m_h < 246$ GeV at 95 % C.L [43, 44]. This assumes that the scale of new physics, Λ , is high enough and thus new physics does not have significant effects on the electroweak precision observables. This upper bound on the Higgs mass may be relaxed if the scale Λ , which suppresses the higher dimensional operators arising from new physics, is around a few TeV [45, 46, 47]. There are also theoretical arguments of triviality and vacuum stability which place bounds on the Higgs mass. A lower bound on the Higgs mass ~ 135 GeV is obtained by requiring the standard model vacuum to be stable to the Planck scale [48, 49, 50, 51]. It was shown in Ref [52, 53, 54, 55] that, in the presence of higher dimensional operators, the vacuum stability limit on the Higgs mass can also be changed. In this work we will study the effects of higher dimensional operators, due to physics around the cut-off, on the Higgs mass prediction in Split Susy and the Little Higgs model. It is worthwhile to justify the addition of higher dimensional operator for vacuum stability analysis. Consider the Split SUSY scenario where the particle spectrum is split with the scalars having masses close to the SUSY breaking scale, m_S . These scalars as well as other possible SUSY multiplet at or around m_S will contribute to the Higgs effective potential and at low energy such effects can be represented by an dimension 6 effective operator. What we point out in the paper is that the effect of this higher dimension operator can be significant for Higgs mass prediction. The same arguments apply to the Little Higgs

model where additional fermions have to be placed around the cut-off to cancel anomalies and for phenomenological reasons. The bottom line is that in any version of the Split SUSY or Little Higgs model the effect of cut-off physics is important and as we show in the paper can produce large corrections to the Higgs mass predictions.

In Ref. [29] the issue of vacuum stability in Split Susy models was addressed by considering the SM vacuum to be not the true vacuum. A constraint on the Higgs coupling can then be obtained at the scale Λ by requiring that the SM vacuum has not decayed to the true vacuum. In our analysis, we consider the SM vacuum to be the true vacuum and require the Higgs potential to have a global minimum at the scale $v \sim 246$ GeV for values of the Higgs field $\lesssim \Lambda$. In our analysis we take into account the effects of the physics at cut-off in the effective potential which was not considered in Ref. [29]. The issue of vacuum stability in a particular Little Higgs model [42] was also discussed but without the effect of physics at cut-off.

The paper is organized in the following manner: in Section. 2 we analyze in general terms the requirement of vacuum stability and the constraints on the Higgs mass. In Section. 3 and Section. 4 we discuss vacuum stability in the Split Susy and a specific Little Higgs model, and finally in Section. 5 we summarize our results.

2 Vacuum Stability- General Analysis

We start with a model that is valid up to some cut-off Λ . Below the scale Λ , are the SM fields as well as additional particles. The physics in this region can be described by an effective theory having the SM gauge symmetry but the lagrangian involves both SM and new particles. The corrections which come from the underlying theory around, and or beyond the cut-off are described by higher dimension operators,

$$\mathcal{L}^{new} = \sum_i \frac{c_i}{\Lambda^{d_i-4}} \mathcal{O}^i, \quad (1)$$

where d_i are the dimensions of \mathcal{O}^i , which are integers greater than 4 and the operators \mathcal{O}^i are $SU(3)_c \times SU(2)_L \times U(1)_Y$ invariant. The dimensionless parameters c_i , determining the strength of the contribution of operators \mathcal{O}^i , can be of $O(1)$ or larger.

In this work, we are interested in an analysis of the Higgs mass bound from consideration of vacuum stability using the effective lagrangian approach with higher dimensional operators. As discussed in Ref.[52, 53], depending on the size and sign of the coupling of the higher dimensional operator, vacuum stability analysis gives a band instead of a single value for the lower bound on the Higgs mass for fixed Λ . In general, Higgs mass bound from vacuum stability complement the bounds obtained from precision electroweak observables. For instance, electroweak precision measurements can be used to obtain an upper bound on the Higgs mass for a given Λ or alternately for a given Higgs mass one can obtain a lower bound on the scale Λ . Vacuum stability analysis provide a lower bound on the Higgs mass for a given Λ and alternately for a given Higgs mass one can obtain an upper bound on the scale of new physics Λ .

Analyses of the higher dimension operators in Eq. 1 have been performed by many authors in the literature[56, 57]. The operator, up to dimension 6, relevant for deriving the lower bound on the Higgs mass from vacuum stability is given by

$$\mathcal{L}^{new} = \frac{c}{\Lambda^2}(\Phi^\dagger\Phi - \frac{v^2}{2})^3, \quad (2)$$

where the Higgs field Φ is

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \phi \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\chi^+ \\ i\chi^0 \end{pmatrix}, \quad (3)$$

and $v = \langle \phi \rangle$ is the vacuum expectation value that minimises the Higgs potential.

The tree level Higgs potential, in the presence of the higher dimensional operator in Eq. 2, can be written as [58]

$$V_{tree} = -\frac{m^2}{2}\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{8}\frac{c}{\Lambda^2}(\phi^2 - v^2)^3, \quad (4)$$

which is corrected by the one-loop term, V_{1loop} ,

$$V_{1loop}(\mu) = \sum_i \frac{n_i}{64\pi^2} M_i^4(\phi) \left[\log \frac{M_i^2(\phi)}{\mu^2} - C_i \right], \quad (5)$$

where

$$M_i^2(\phi) = k_i \phi^2 - k'_i. \quad (6)$$

The summation goes over the gauge bosons, the fermions and the scalars of the standard model. The values of the constants n_i , k_i , k'_i and C_i can be found in Refs[48, 49, 50, 51, 59]. The full effective potential up to one-loop correction is

$$V = V_{tree} + V_{1loop}. \quad (7)$$

To obtain a lower bound on the Higgs boson mass, in the absence of higher dimensional operators, one can take the location of vacuum instability to be as large as Λ . However, in our approach, for the low energy theory to make sense ¹, we should require $\phi < \Lambda$. We take the scale of vacuum instability, Λ' , to be 0.5Λ , so the corrections from operators of dimension greater than six to our result is suppressed by a factor of $\frac{\Lambda'^2}{\Lambda^2} = 0.25$.

Since we are dealing with values of the field ϕ larger than v , we need to consider a renormalization group improved potential for our analysis [48, 49, 50, 51, 59, 60, 61, 62, 63]. As indicated earlier, we have below the scale Λ , not only the SM fields but additional new particles. Hence running for the Higgs self coupling, λ , the top Yukawa coupling (g_Y) will be modified. Since the effective potential contains one loop correction from the SM fields we use 2-loop running in the beta functions as far as the SM contributions are involved [64, 65]. For the new particles, their contributions to the beta functions are considered at the one-loop level and the loop effects of the new particles to the effective

¹Effective theory in general will not be valid in the region close to the cutoff scale. One of the examples is the chiral lagrangian of pions where the predictions for processes such as $\pi\pi$ scattering can only be reliable for small momentum transfer relative to the cutoff $\Lambda \sim 4\pi f_\pi \sim 1$ GeV.

potential is neglected. This is not unreasonable as the new particles are higher in mass than the SM particles and the one loop correction from them to the effective potential will be smaller than the SM contribution. The various β functions for the SM running to two-loop order can be found in Ref [66].

In the presence of higher dimensional operators, with the scale of vacuum stability Λ' , the vacuum stability requirement

$$V(\Lambda') = V(v) \quad (8)$$

provides the boundary condition for λ at the scale Λ' , which using Eq. 7 and Eq. 8 is given by

$$\lambda_{eff}(\Lambda') \approx -\sum_i \frac{n_i}{16\pi^2} k_i^2 (\log k_i - C_i) - \frac{1}{2} \frac{\Lambda'^2}{\Lambda^2} c + \frac{2m^2}{\Lambda'^2}, \quad (9)$$

where

$$\lambda_{eff}(\Lambda') = \lambda(\Lambda') - \frac{3}{2} c \frac{v^2}{\Lambda^2}. \quad (10)$$

One can then run down the Higgs coupling λ to obtain the Higgs mass that ensures vacuum stability to the scale Λ' . In general the effective potential increases with ϕ for $\phi > v$ and attains a local maximum beyond which it turns around and becomes unstable at the scale Λ' where the depth of the potential is the same as for $\phi = v$.

Note that the effect of a positive c , in Eq. 2, is to delay the onset of vacuum instability compared to the standard model while the effect of a negative c is to accelerate the onset of vacuum instability. For large enough positive values of c , the effect of the higher dimensional operator can compensate for the tendency of the standard model Higgs potential to become unstable and the instability of the effective potential disappears for all values of $v \leq \phi \leq \Lambda'$. This effect can be demonstrated in a toy model of new physics where a scalar field of mass M is added to the standard model [67]. It was shown in Ref [67] that, for a given choice of parameters in the effective potential, there is a critical value for the scalar mass M , below which the vacuum instability disappears.

In such cases the boundary condition for λ at the scale Λ' , is no longer given by Eq. 8 and one has to numerically search for the minimum Higgs mass that ensures

$$V(\phi) \geq V(v) \quad (11)$$

for all $\phi \leq \Lambda'$. In fact if one starts from the boundary condition of Eq. (6) the vacuum becomes unstable much before Λ' and the potential attains a second local minimum which is deeper than the minimum at $\phi = v$ [53].

3 Vacuum Stability- Split Susy

In the Split Susy scenario the scale of susy breaking $m_S \equiv \Lambda$ is very high, well beyond a TeV. Below the scale of SUSY breaking, are the fermion superpartners, the Higgsino $\tilde{H}_{u,d}$, the gluino (\tilde{g}), the W-ino (\tilde{W}) and B-ino (\tilde{B}), and the SM particles with one Higgs doublet. Following Ref. [5], the most general renormalizable Lagrangian, relevant to our study, with a matter parity, and gauge-invariant kinetic terms, is given by

$$\begin{aligned} \mathcal{L} = & m^2 H^\dagger H - \frac{\lambda}{2} (H^\dagger H)^2 - \left[h_{ij}^u \bar{q}_j u_i \epsilon H^* + h_{ij}^d \bar{q}_j d_i H + h_{ij}^e \bar{\ell}_j e_i H \right. \\ & + \frac{M_3}{2} \tilde{g}^A \tilde{g}^A + \frac{M_2}{2} \tilde{W}^a \tilde{W}^a + \frac{M_1}{2} \tilde{B} \tilde{B} + \mu \tilde{H}_u^T \epsilon \tilde{H}_d \\ & \left. + \frac{H^\dagger}{\sqrt{2}} (\tilde{g}_u \sigma^a \tilde{W}^a + \tilde{g}'_u \tilde{B}) \tilde{H}_u + \frac{H^T \epsilon}{\sqrt{2}} (-\tilde{g}_d \sigma^a \tilde{W}^a + \tilde{g}'_d \tilde{B}) \tilde{H}_d + \text{h.c.} \right], \end{aligned} \quad (12)$$

where $\epsilon = i\sigma_2$.

The Lagrangian in Eq. 12 now should be matched to the fully supersymmetric SM at the scale Λ which includes the scalars- the squarks, sleptons, charged and pseudoscalar Higgs and is given by

$$\begin{aligned} \mathcal{L}_{\text{susy}} = & -\frac{g^2}{8} (H_u^\dagger \sigma^a H_u + H_d^\dagger \sigma^a H_d)^2 - \frac{g'^2}{8} (H_u^\dagger H_u - H_d^\dagger H_d)^2 \\ & + \lambda_{ij}^u H_u^T \epsilon \bar{u}_i q_j - \lambda_{ij}^d H_d^T \epsilon \bar{d}_i q_j - \lambda_{ij}^e H_e^T \epsilon \bar{e}_i \ell_j \\ & - \frac{H_u^\dagger}{\sqrt{2}} (g \sigma^a \tilde{W}^a + g' \tilde{B}) \tilde{H}_u - \frac{H_d^\dagger}{\sqrt{2}} (g \sigma^a \tilde{W}^a - g' \tilde{B}) \tilde{H}_d + \text{h.c.} \end{aligned} \quad (13)$$

The matching allows us to obtain the coupling constants of the effective theory at the scale Λ . These are given by [5]

$$\lambda(\Lambda) = \frac{[g^2(\Lambda) + g'^2(\Lambda)]}{4} \cos^2 2\beta, \quad (14)$$

$$\begin{aligned} h_{ij}^u(\Lambda) &= \lambda_{ij}^{u*}(\Lambda) \sin \beta, & h_{ij}^{d,e}(\Lambda) &= \lambda_{ij}^{d,e*}(\Lambda) \cos \beta, \\ \tilde{g}_u(\Lambda) &= g(\Lambda) \sin \beta, & \tilde{g}_d(\Lambda) &= g(\Lambda) \cos \beta, \\ \tilde{g}'_u(\Lambda) &= g'(\Lambda) \sin \beta, & \tilde{g}'_d(\Lambda) &= g'(\Lambda) \cos \beta. \end{aligned} \quad (15)$$

The h_{ij} 's and the λ_{ij} 's are the Yukawa couplings in the effective theory (Eq. 12) and in the full theory (Eq. 13). The Higgs doublet, H in Eq. 12 is given by $H = -\cos \beta \epsilon H_d^* + \sin \beta H_u$ where $\tan \beta = v_1/v_2$ with $v_{1,2}$ being the v.e.v of the neutral component of the Higgs doublet $H_{u(d)}$ satisfying $v = \sqrt{v_1^2 + v_2^2} \sim 246$ GeV.

As pointed in Ref. [29] the value of the Higgs coupling, λ , at the scale Λ can be easily modified in the presence of additional terms in the theory at the scale Λ . In our analysis the boundary condition for the Higgs coupling at the scale Λ' , the scale of vacuum stability, will be provided by the requirement of vacuum stability in Eq. 8. The boundary conditions for the other coupling will be taken from Eq. 15.

In Fig. 1 and Fig. 2 we plot the Higgs mass versus the scale of SUSY breaking, Λ . As we see from the figure the effect of the higher dimensional operator has a significant effect on the Higgs mass specially if the sign of c is negative. Hence, in the presence of higher dimensional operators, the Split Susy model can allow a much larger Higgs mass, well beyond the Higgs mass range. quoted in the literature[5, 29]. For $c = 0$ our predictions for the Higgs mass is similar to that obtained in Ref. [29]. Notice that the Higgs mass for vacuum stability in this case ($c=0$) is bigger than in the SM, where a Higgs mass of about 135 GeV ensures vacuum stability up to the Planck scale[48, 49, 50, 51]. This is because of additional fermionic fields that modify the running of the Higgs coupling.

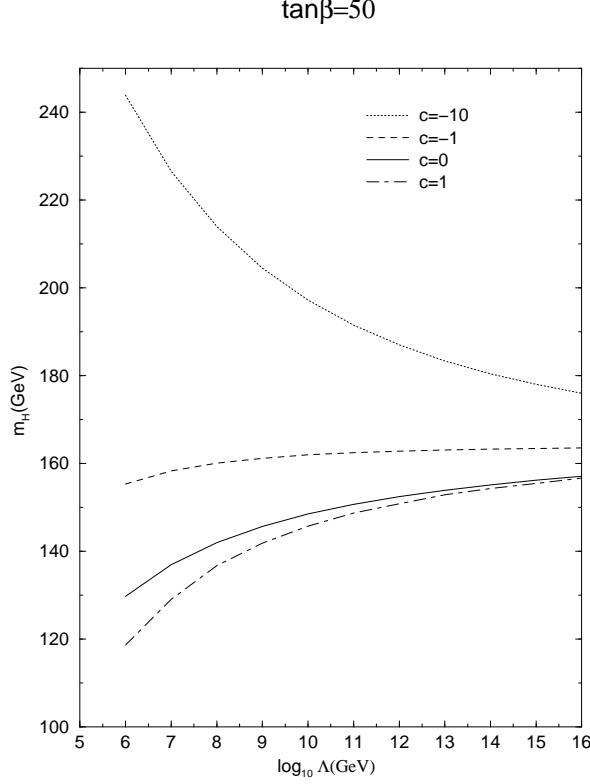


Figure 1: The Higgs mass versus the scale of new physics Λ for $\tan\beta=50$ in Split Susy

4 Vacuum Stability- Little Higgs Model

One of the attractive features of the Little Higgs models is a natural explanation of electroweak symmetry breaking because of the large top Yukawa coupling. Radiative correction involving a $SU(2)_L$ singlet heavy quark ,T, can provide a negative mass-squared contribution to the Higgs potential thereby triggering electroweak symmetry breaking. There are various versions of the Little Higgs model and we will consider a particular simple version that was proposed in Ref. [42]. In this simple $SU(3)$ model the weak interactions of the Standard Model are enlarged from $SU(2) \times U(1)$ to $SU(3) \times U(1)$. There are new gauge bosons associated with the enlarged gauge symmetry and a new heavy top quark. The quadratic divergences to the Higgs mass from the SM gauge bosons and the top quark are canceled by the new gauge bosons and the heavy top quark. In

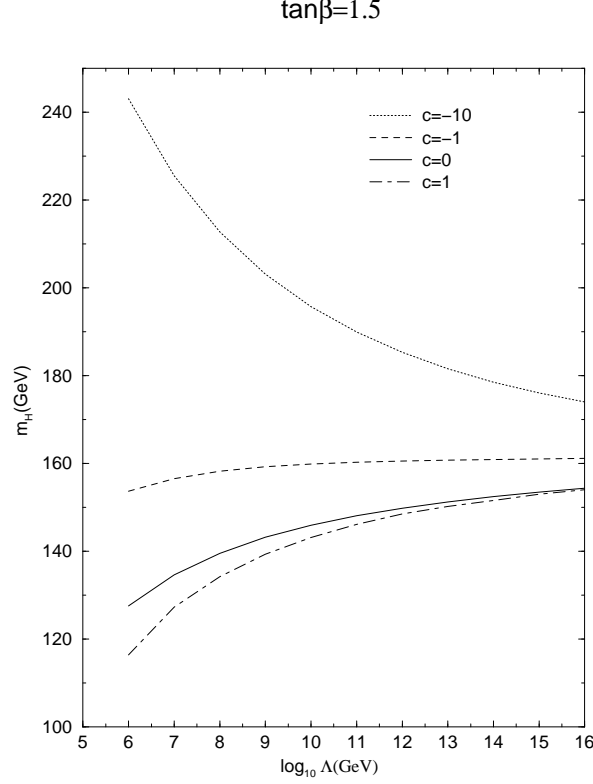


Figure 2: The Higgs mass versus the scale of new physics Λ for $\tan\beta=1.5$ in Split Susy

this model, besides a tree level potential term, the other terms of the Higgs potential are generated dynamically through radiative corrections. The details of this model are presented in Ref. [42] and here we just provide the bare essentials for the vacuum stability analysis.

The SM gauge group $SU(2)_w \times U(1)_Y$ is enlarged to $SU(3)_w \times U(1)_X$ and the symmetry breaking, $SU(3)_w \times U(1)_X \rightarrow SU(2)_w \times U(1)_Y$, occurs when two complex triplet scalar fields Φ_1 and Φ_2 that transform as $(1, 3)_{-\frac{1}{3}}$ under $(SU(3)_c, SU(3)_w)_{U(1)_X}$ get vevs f_1 and f_2 . After the breaking of $SU(3)_w$ the remaining degrees of freedom in the Φ_i are conveniently parametrized in the non-linear representation of the gauge symmetry as

$$\Phi_1 = e^{i\Theta \frac{f_2}{f_1}} \begin{pmatrix} 0 \\ 0 \\ f_1 \end{pmatrix}, \quad \Phi_2 = e^{-i\Theta \frac{f_1}{f_2}} \begin{pmatrix} 0 \\ 0 \\ f_2 \end{pmatrix} \quad (16)$$

where

$$\Theta = \frac{1}{f} \left[\frac{\eta}{\sqrt{2}} + \begin{pmatrix} 0 & 0 & h \\ 0 & 0 & \\ h^\dagger & 0 & \end{pmatrix} \right] \quad \text{and} \quad f^2 = f_1^2 + f_2^2. \quad (17)$$

The field h is identified with the SM Higgs doublet Φ defined in Eq. 3 and η is a real scalar field.

The masses for the gauge bosons, W_\pm and the $SU(2)$ doublet of heavy gauge bosons (W'_\pm, W'_0) are given by

$$\begin{aligned} m_{W_\pm}^2 &= \frac{g^2}{4} v^2 \\ m_{W'_\pm}^2 &= m_{W'_0}^2 = \frac{g^2}{2} f^2, \end{aligned} \quad (18)$$

where $f^2 = f_1^2 + f_2^2$. The neutral gauge bosons have masses

$$\begin{aligned} m_Z^2 &= \frac{g^2}{4} v^2 (1 + t^2) \\ m_{Z'}^2 &= g^2 f^2 \frac{2}{3 - t^2} \end{aligned} \quad (19)$$

where $t = g'/g = \tan \theta_W$ and θ_W is the weak mixing angle.

There are various ways in which the fermions of the theory can be made to transform under the enlarged gauge group. The masses and mixing of the quarks arise from Yukawa interactions and the form of the interactions depend on how one chooses to transform the quarks under $(SU(3)_c, SU(3)_w)_{U(1)_X}$. Within a fairly general scenario that does not allow tree level flavour changing neutral current effects one obtains for the masses of the up-type quarks and their partners as

$$\begin{aligned} m_u &= \lambda_u \langle h \rangle \\ m_U &= \sqrt{(\lambda_1^u f_1)^2 + (\lambda_2^u f_2)^2} \end{aligned} \quad (20)$$

where

$$\lambda_u = \lambda_1^u \lambda_2^u \frac{f}{m_U}, \quad (21)$$

and λ_1^u and λ_2^u are Yukawa couplings. Of interest to us is the mass of the heavy top, m_T , as it plays an important role in electroweak symmetry breaking. The mass of the m_T depends on unknown Yukawa couplings, and so to reduce the number of parameters, following Ref. [42], we determine the λ_i^t such that the T -mass is minimized for given scales f_i . This gives the values

$$\begin{aligned}\lambda_1 &= \sqrt{2}\lambda_t \frac{f_2}{f} \\ \lambda_2 &= \sqrt{2}\lambda_t \frac{f_1}{f},\end{aligned}\tag{22}$$

where λ_t is the top Yukawa coupling. The top and the T quark masses are then given by, using Eq. 20

$$\begin{aligned}m_t &= \lambda_t \frac{v}{\sqrt{2}} \\ m_T &= 2\lambda_t \frac{f_1 f_2}{f},\end{aligned}\tag{23}$$

with $v \sim 246$ GeV is the Higgs vacuum expectations value.

For the study of vacuum stability, we do not expect that a more general expression for m_T in Eq. 20 will significantly alter our results.

The tree level effective potential in this model comes from

$$V_{tree} = \mu^2 \Phi_1^\dagger \Phi_2 + h.c. \rightarrow \mu^2 \frac{f^2}{f_1 f_2} (h^\dagger h + \frac{1}{2} \eta^2) - \frac{1}{12} \frac{\mu^2 f^4}{f_1^3 f_2^3} (h^\dagger h)^2 + \dots\tag{24}$$

which follows from expanding the $\Phi_{1,2}$ field using Eq. 16 and Eq. 17. Including the radiative corrections and replacing h by $h \equiv \Phi$ defined in Eq. 3, the effective Higgs potential is given by

$$\begin{aligned}V_{eff} &= -\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 \\ m^2 &= -(\mu^2 \frac{f^2}{f_1 f_2} + \delta m^2) \\ \lambda &= (-\frac{1}{12} \frac{\mu^2 f^4}{f_1^3 f_2^3} + \delta\lambda)\end{aligned}\tag{25}$$

where

$$\begin{aligned}
\delta m^2 &= \frac{-3}{8\pi^2} \left[\lambda_t^2 m_T^2 \text{Log} \left(\frac{\Lambda^2}{m_T^2} \right) - \frac{g^2}{4} m_{W'}^2 \text{Log} \left(\frac{\Lambda^2}{m_{W'}^2} \right) - \frac{g^2}{8} (1+t^2) m_{Z'}^2 \text{Log} \left(\frac{\Lambda^2}{m_{Z'}^2} \right) \right] \\
\delta \lambda &= \frac{3}{16\pi^2} \left[\lambda_t^4 \text{Log} \left(\frac{m_T^2}{m_t^2} \right) - \frac{g^4}{8} \text{Log} \left(\frac{m_{W'}^2}{m_W^2} \right) - \frac{g^4}{16} (1+t^2)^2 \text{Log} \left(\frac{m_{Z'}^2}{m_Z^2} \right) \right] \\
&+ \frac{|\delta m^2|}{3} \frac{f^2}{f_1^2 f_2^2}
\end{aligned} \tag{26}$$

The condition $(\frac{dV_{eff}}{d\phi})_v = 0$ then gives

$$\mu^2 = \frac{12f_1^3 f_2^3 (\delta m^2 + v^2 \delta \lambda)}{f^2 (-12f_1^2 f_2^2 + v^2 f^2)} \tag{27}$$

Now the Higgs effective potential V_{eff} in Eq. 25 is determined in terms of the parameters, $f_{1,2}$ and Λ . The cut-off, Λ is $\sim 4\pi f_1$ [42] so we will choose $\Lambda = 4\pi f_1$ to further reduce the parameter space of the model. Hence for a given value of Λ , f_1 is fixed. For a given f_1 and m_T the Higgs mass is fixed. For the choice $m_T = 1$ TeV and $f_1 = 0.5$ TeV the Higgs mass is around 140 GeV. In our analysis, we will vary Λ between 5-10 TeV. Using Eq. 23 we can now solve for f in terms of m_T ,

$$f = \frac{2f_1^2 \lambda_t}{\sqrt{4f_1^2 \lambda_t^2 - m_T^2}}. \tag{28}$$

We will consider $f \geq 2$ TeV to satisfy the electroweak precision measurements[42]. As f is real, we have the constraint from Eq. 28,

$$m_T \leq 2\lambda_t f_1. \tag{29}$$

Hence for a given Λ and m_T , subject to the constraint in Eq. 29, the effective potential is completely determined and so is the Higgs mass.

Now we add the effect of physics at or around the cut-off, Λ , via a higher dimensional operator to the Higgs potential in Eq. 25. The little Higgs model matches on to the SM at the scale f below which the gauge structure of the model is SM. Above the scale f up to the cut off Λ the symmetry

of the model is $(SU(3)_c, SU(3)_w)_{U(1)_X}$. The physics at the cut-off can generate contributions to the effective potential invariant under $(SU(3)_c, SU(3)_w)_{U(1)_X}$ of the form

$$V_\Lambda = a_1 M^2 (\Phi_1^\dagger \Phi_2) + a_2 (\Phi_1^\dagger \Phi_2)^2 + a_3 (\Phi_1^\dagger \Phi_2)^3 / \Lambda^2 .. + h.c, \quad (30)$$

where M is some mass parameter and V_Λ is the contribution to the effective potential from physics at the cut-off Λ . Writing out V_Λ explicitly in terms of the SM Higgs field ϕ we can write

$$V_\Lambda = Am^2 \phi^2 + B\phi^4 + C\phi^6 / \Lambda^2. \quad (31)$$

where m is another mass parameter and the various co-efficients A, B and C are some combination of $a_{1,2,3}$ and $f_{1,2}$ with $\Lambda = 4\pi f_1$.

Now using the conditions

$$\begin{aligned} \left(\frac{dV_{eff}}{d\phi} \right)_v &= 0 \\ \left(\frac{dV_{eff}^2}{d\phi^2} \right)_v &= m_H^2, \end{aligned} \quad (32)$$

allows us to recast V_Λ in form of the higher dimensional contribution to the effective potential in Eq. 2. The full effective potential is then written as,

$$V_{eff} = -\frac{m^2}{2} \phi^2 + \frac{1}{4} \lambda \phi^4 + \frac{1}{8} \frac{c}{\Lambda^2} (\phi^2 - v^2)^3, \quad (33)$$

The effective potential above should be valid for values of $\phi \leq f$. For a given value of the cut-off Λ , as we vary m_T in Eq. 28, there is a maximum value for f which we call f_{max} (Note that $f \geq 2$ TeV to satisfy the electroweak precision constraints). The value for f_{max} is greater than $\Lambda/2$ for values to $\Lambda = 5 - 10$ TeV. Keeping only terms to order dimension six to represent the effects of cut-off make sense for values of ϕ sufficiently less than Λ and so like the vacuum stability analysis for Split Susy we will choose $\Lambda' = 0.5\Lambda$ to be the scale of vacuum stability. Note that $\Lambda' \sim f_{max}$ and so the form of Eq. 33 to represent the effects of physics around cut-off is justified.

The requirement of vacuum stability then gives

$$\begin{aligned}\lambda(m_T, \Lambda) &= -\frac{c}{2} \frac{(-\Lambda'^2 + v^2)^3}{\Lambda^2 \Lambda'^2 (2v^2 - \Lambda'^2)} \\ &\sim -\frac{c}{2} \frac{\Lambda'^2}{\Lambda^2}\end{aligned}\tag{34}$$

where Λ' is the scale of vacuum stability discussed above.

As $\lambda(m_T, \Lambda)$ is positive, therefore for positive c the vacuum is always stable up to the scale Λ' . For negative values of c , $\lambda(m_T, \Lambda) \geq \frac{|c|}{2} \frac{\Lambda'^2}{\Lambda^2}$ to ensure vacuum stability. In Fig. 3 we plot the Higgs

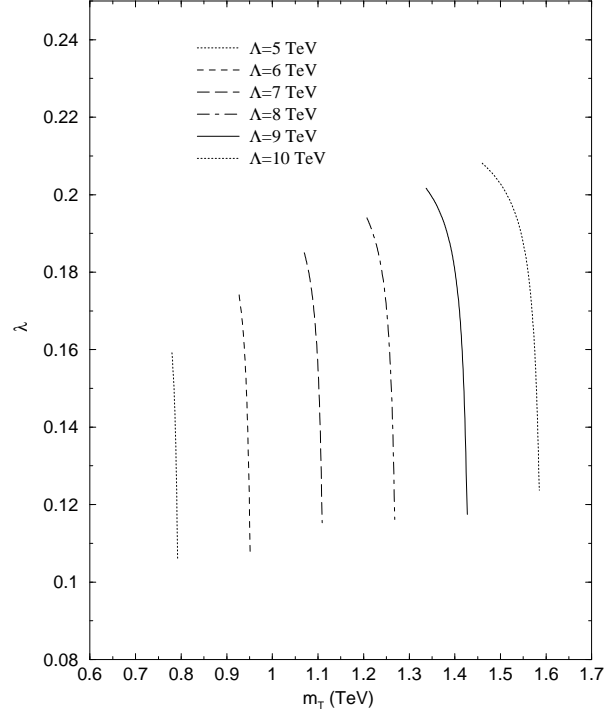


Figure 3: The Higgs coupling λ versus m_T for various Λ

coupling versus m_T for various Λ . In particular, for $\Lambda = 10$ TeV the maximum Higgs coupling, $\lambda_{max} \sim 0.21$ which corresponds to a Higgs mass, $m_h = \sqrt{2\lambda}v \sim 160$ GeV. This corresponds to $c \sim 1.7$ from Eq. 34 with the scale of vacuum stability $\Lambda' = 0.5\Lambda$. Hence for negative values of c , with $|c| > 1.7$ the vacuum will become unstable. For lower values of Λ and or m_T , smaller $|c|$ will

destabilize the vacuum. Hence, for the Little Higgs model to be viable the corrections from higher dimensional operators have to be suppressed sufficiently. While we have worked in a specific model our conclusion is going to be true for different models also, simply due to the fact that the cut-off $\Lambda \sim 10$ TeV or less is low and the effect from higher dimensional operators can be quite significant on the effective potential and Higgs mass prediction.

5 Summary

In summary, we have studied the effect of higher dimensional operators in the effective potential in two classes of models- the Split Susy models and the Little Higgs models. In both cases the effects of the higher dimensional operators is important when it comes with a negative coefficient. In Split Susy models the Higgs mass predictions can be changed significantly specially when the cut-off scale is relatively low. In the Little Higgs models where the cut-off scale is much lower, the effect of higher dimensional operator is significant. In a particular Little Higgs model a value of the coefficient of the higher dimensional operator of $O(1)$ can destabilize the vacuum.

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